A PRE-ANALYSIS OF THE CREATION OF TEACHER RESOURCES FOR DEVELOPING INSTRUCTION IN BASIC LOGIC IN FRENCH HIGH SCHOOLS

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12th International Congress on Mathematical Education
Program Name XX-YY-zz (pp. abcde-fghij)
8 July – 15 July, 2012, COEX, Seoul, Korea (This part is for LOC use only. Please do not change this part.)

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If everyone agrees that logic is needed to do mathematics, there are divergences concerning the role of mathematical logic in acquiring the necessary and sufficient knowledge in this area. We will try first to see what might be the students' difficulties in the acquisition of logic of mathematics and what can be Mathematical Knowledge for Teaching Logic of mathematics. A study of syllabuses and textbooks for high school in France shows strong constraints and ill-defined conditions for this teaching. In their answers to a questionnaire we proposed, some teachers expressed their lack of theoretical knowledge in mathematical logic and lack of resources to present to their pupils activities in order to address notions of logic. During a continuous training, we try to offer an approach of mathematical logic which support teaching of logic of mathematics.

Logic -Mathematical Knowledge for Teaching-Language-Reasoning

INTRODUCTION

In the introduction to the proceedings of the ICMI Study 19 Conference : Proof and Proving in Mathematics Education (2009), the authors note that some research should be pursued to understand the role of logic in teaching proof. The experience of teaching formal logic in high school during the time of “modern mathematics” in the 1970s in different countries has shown that this approach does not directly provide students with effective tools to improve their abilities in expression and reasoning. This observation leads us to think further about the following question: how and why should logic be introduced in a math class?

In France, logic was not part of the high school curriculum from 1981 to 2001. It was re-introduced in the 2001 syllabus and associated with work on reasoning. The 2009 syllabus goes even further: it includes objectives for "mathematical notations and reasoning" which are linked to notions which depend on mathematical logic. It is for us an interesting context to debate the issue of logic in math class, and we are particularly interested[1] in investigating how teachers implement in their classrooms activities to introduce the concepts of logic and achieve the goals of the new program.

[1] It is the subject of a thesis in mathematics education, started in September 2010 under the direction of Michèle Artigue, René Cori and Cécile Ouvrier-Buffet.

First of all, we will present few epistemological analysis about the links between logic and mathematics. These analysis will be used further on to comment different researches on the role of logic in the teaching of mathematics. Some of these researches study the logical dimension found in students' work, and show the difficulties students have to prove or reformulate statement that have a complex logical structure. Other researches rather focus on the Mathematical Knowledge for Teaching. Therefore we will see some of the specificities of logic, both from the point of view of subject matter knowledge and didactical knowledge. These specificities reveal the complexity in the relationship between logic and mathematics.

We will use the reflection concerning these researches as a background to broach more specific matters. The first one is to understand what "teaching logic" means for the teachers, and what notions are at stake in learning. We ask this question in a particular context: the one of the High schools in France, and more precisely under the effect of the new syllabus. We have then first analysed syllabuses and high school French textbooks since 1960. This allows us to study the first part of the didactic transposition (Chevallard, 1985): the passage of scholarly knowledge ("savoir savant") to knowledge to be taught ("savoir à enseigner"). These documents tell us about the conditions and the constraints determining a context of teaching logic for the teachers. We will present after that the results of a questionnaire we have elaborated, and that give us some elements of answer on the reactions of the teachers to this context. These results allow us to extract some features about their didactical choices for this teaching. The answers we got confirm the difficulties to actually put into practice a teaching of notions of logic, which leads us to another question: what training is to be given to the teachers so they can succeed in that teaching? We will present a training course, "initiation to logic", offered to teachers in activity, and we will explain the choices concerning the subjects of this training course.

STUDIES ON THE ROLE OF LOGIC IN MATHEMATICS EDUCATION.

Some considerations on the epistemology of logic.

The study field of logic seen as the science of reasoning largely goes beyond the scope of mathematics. Even within this discipline, we will distinguish between the mathematical logic on one hand, which is a recent branch of mathematics, and the logic of mathematics on the other hand, that we will define as the art of organizing one's speech in that discipline, seen under the double aspect of syntactic correction and semantic validity. We will call logical knowledge all knowledge being a matter of this art.

If the mathematical logic has been, among others, constituted in the purpose of modeling the logic of mathematics, a common word amongst mathematicians is that it is not necessary to use the mathematical logic to broach the logic of mathematics. We will therefore have to also distinguish between the fact of giving a course of mathematical logic, which means a course of mathematics in which we will be studying objects as formulas, connectives, quantifiers, models… and teaching the logic of mathematics, which means transmitting to the students the way of using some useful rules to reason and express oneself in mathematics. In that second case, we will also speak of teaching notions of logic.

Let's consider for instance something well known in mathematics: the negative of a universal statement is an existential statement. This formulation is too imprecise to pretend being other than a shortcut. To get things more precise, we need to do a first work of formalization: define what a statement is, or a proposition, and define the attitude towards the truth values of the connectives and quantifiers that operate on these propositions. In contemporary mathematical terms, we will state the following rule, that is part of the logic of mathematics:

Rule 1: The negative of the proposition "for all x, P(x)" is the proposition "there is at least one x such as nonP(x)".

The mathematical logic will make one more step in the formalization in "mathematizing" the notions at work. Rule 1 will then become a theorem:

Theorem 1: the formula $\neg(\forall x\ P(x))$ is logically equivalent to the formula $\exists x\ \neg\ P(x)$.

This example allows us to make a final distinction concerning the language this time, between the formal language of predicates, studied among other formal languages by logicians (formulas such as the ones in theorem 1 are the "sentences" of this language), and the mathematical language used by mathematicians who surf following their needs among formulations more or less formalized, which means obeying to a formatting more or less close to the syntax of the formal language of predicates.

These terminological precisions will now be useful to us to comment existing researches on the question of logic in the teaching of mathematics.

Comments on student activities.

Various studies based on experiments with university students show the difficulties they have in understanding and proving quantified statements (Dubinsky, Yiparaki, 2000, Arsac, Durand-Guerrier, 2003, Chellougui, 2009, Roh, 2010). For most students engaged in proving if a statement is true or false, the relationship between the quantified formulation of a statement and the framework of its proof is not clear. Thus, while recognizing the role of informal statements in memorizing mathematical results, J. Selden and A. Selden make the assumption that the ability to unpack the logic of an utterance by writing it formally is related to the ability to ensure the validity of a proof of this statement (J. Selden and A. Selden, 1995). These authors suggest that students are accustomed since high school to providing proofs and to reflecting on their own actions.

Moreover, a reformulation work of mathematic wordings is often necessary, because the logic of mathematics' rules, such as rule 1 seen before, fit over statements whose structure is close to the syntax of the formal language of predicates, in other words, fit over statements whose logical structure has been unpacked. We rather use the adjective "formal" for what is part of the mathematical logic. Therefore we will call such statements "formalized statements", which doesn' mean "symbolic", but only "in conformity with certain formatting rules". For example, a rule, linked to the expression of the formulas in the formal language of predicates, requires that the quantification of a variable be explicit before this variable appears for the first time. So to say "u and v are relatively prime", we may, with the Bezout identity, say "au + bv = 1, where a and b are integers", but this formulation does not respect the rule of explicitness of quantifications. However, this statement is synonymous with "there exists an integer a and an integer b such that au + bv = 1", which respects the rule. Students who need to write the negation of the statement "u and v are relatively prime" may be puzzled by the sentence "au + bv = 1, where a and b are integers" whose form doesn't allow a direct application of the formation rules of negative. Such tasks of reformulation at different levels of language can be offered in high school using simple statements. For example, "y has a pre-image under the function f" is reformulated as "there exists at least one x such that y = f(x)" or, conversely, "for all real x, x2 ≥ 0" is reformulated as "a square is always positive". This allows students to work on statements along a syntactic dimension (study of their forms) and a semantic dimension (study of their meanings). Another work consists of reformulating a statement in different registers. Stephanie Bridoux (Bridoux, 2009) shows how the connection between different registers of language, through reformulation tasks, allows students to make sense of the formal definitions of topology in the first year of university courses. We believe that these two kinds of reformulation work are important because they contribute to the construction of what J. Selden and A. Selden call "statement images" (J. Selden and A. Selden, 1995).

The studies evoked in this paragraph allow us to think of various possible activities for students, in which notions of logic play a part, and show certain of the difficulties the students meet. We are now going to discuss the question of which Mathematical Knowledge should the teachers have to offer such kind of activities, and more generally to teach notions of logic.

Comments on Mathematical Knowledge for Teaching logic.

The study, even minimal, of mathematical logic isn't part of the French classic degree courses of studying mathematics. Nevertheless, we do defend its place as a reference mathematical theory in order to teach the logic of mathematics, which isn't the case actually. Using the description of the MKT given by D.L Ball, H. Thames and G Phelps (Ball, Thames and Phelps, 2008) shown in figure 1, we can see this lack of a reference mathematical theory as a specificity of the common content knowledge concerning logic. Moreover, we can assume that the specialized content knowledge, as well as the horizon content knowledge, will be relatively poor because only a few teachers would have followed mathematical logic courses in their initial training. Some researches evoked by A.J Stylianides and D.L Ball (Stylianides and Ball, 2008) show the lack of knowledge of teachers about the logical linguistically structure of proofs.

The teachers knowledge in order to teach logic of mathematics will then essentially be constituted of what they would have extracted of their mathematical practice. This one practice can give notions of logic that will be more or less well used tools, but for us, it doesn't allow to give theses notions a status of mathematical objects. How can we then imagine an adapted didactical transposition so they become teaching objects? This didactical transposition should consider the transversal character of the logic of mathematics. This transversal character implies that the teachers' knowledge in the field of logic be particularly available so they can be used at any favorable moment in the mathematical activity.

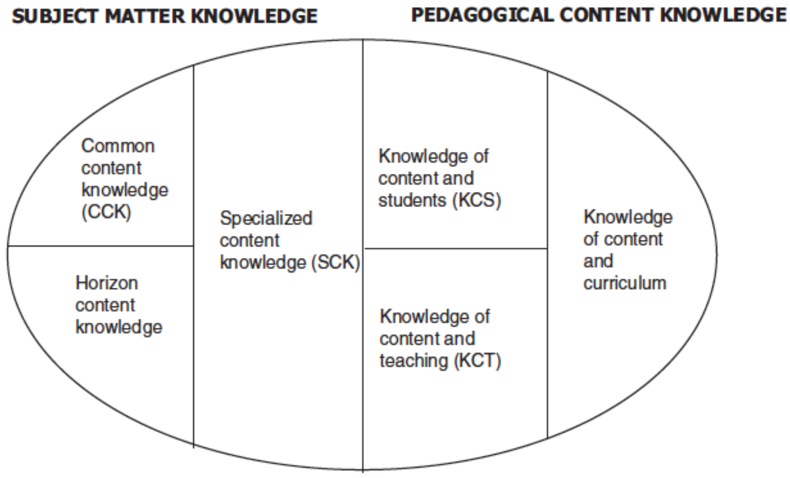

Figure 1 : Domains of MKT (Ball, Thames and Phelps, 2008)

On another hand, concerning the didactical knowledge of teachers, this transversal character implies that they can clearly identify the situations that could be teaching notions of logic ones, and the notions that are at work in a situation or another. We have mentioned in the first part of our article that the proof situations and the reformulations ones are seen by researchers as favorable situations to teach the logic of mathematics. But having been confronted to such situations in ones mathematical career isn't always enough to know how logical knowledge come on top of such situations. Moreover, reformulation activities aren't generally present as such in the mathematical activity, and might then remain invisible for many mathematicians, such as the implicit quantification practice associated to the "if.. then.." statement.

We can see that we cannot gather from the teachers' lack of training in mathematical logic that there is a lack of knowledge to teach the logic of mathematics. But we make the hypothesis that such a training might make their teaching easier. But before discussing about such training, we want to present a beginning of a study about practices of teachers that have to transmit as they can notions of logic to their students.

TEACHING LOGIC OF MATHEMATICS IN HIGHSCHOOL IN FRANCE.

Logic in syllabuses and highschool textbooks in France since 1960.

The study of syllabuses and highschool textbooks allow us first to determine the space of conditions and constraints for the teacher, concerning the teaching of logic of mathematics. Concepts of logic are mentioned for the first time in the mathematics curriculum for students in their first year of high school in 1960. During the middle of the twentieth century, the French mathematicians were strongly influenced by the Bourbaki group of mathematicians, whose axiomatic style spread into teaching. The reform called "modern mathematics" came into force in high school with the syllabus for first-year high school students in 1969. This syllabus was based on the idea of a unified mathematic, that could be used in experimental sciences as well as in human ones. Mastering the language of mathematics was then essential. All texbooks at this time start with a first chapter on set theory and logic, which are the foundations of this language. This period, during which students had a "course in logic", despite the instructions that specified that this should not be done in a dogmatic manner, is still often cited as an example of excessive formalism. And many teachers had difficulty in explaining this formal logic, as well as its relations with language and reasoning, simply because they had not themselves been trained in this domain. In 1981 came a new syllabus, radically different. It was the time of the "counter-reform", in which logic is explicitly excluded from mathematics teaching. This lasted until the implementation of the 2001 syllabus, which states that "training in logic is part of the requirements of high school classes"[2]. This text is included in the 2009 syllabus and is supplemented by a table setting targets for "notations and mathematical reasoning". These objectives relate to certain objects of mathematical logic, but are rather vague: "the students are trained on examples" to properly use connectives and quantifiers, but also to use different kinds of reasoning.

Textbooks authors should follow the directions, even those imprecise given in the syllabus, to provide teachers with tasks for their students. Thus, in most textbooks published in September 2010, we can find pages offering a brief overview of the concepts of logic mentioned in the syllabus. These pages are not a separate chapter, but rather a sort of reference lexicon. We can also find in various chapters exercises marked "logic". By offering a "dressed knowledge" ("savoir apprêté") (Ravel, 2003), textbooks are involved in the second moment of the didactic transposition: the transition of knowledge to be taught (savoir à enseigner"), as defined in the texts, to taught knowledge ("savoir enseigné"), as it is done in the classrooms. An analysis of ten mathematics textbooks published in 2010 shows a diversity in the presentations. Some books have one approach that can be called "propositional", that is, they constitute a kind of "grammar of mathematical propositions". Other books have an approach which can be called "natural", that is, that they take common language as the starting point for the construction of mathematical language, while specifying the requirements for this discipline, in particular, the requirements of univocal meaning for each word.

This analysis of curricula and textbooks shows that these documents do not constitute a reference for teachers that gives them clear guidance on the concepts involved and how to teach. And the lack of knowledge in logic of mathematics can weaken their critical analysis capacity of these documents.

About the desires and needs expressed by teacher: some results of a questionnaire

As we said, we have elaborated a questionnaire to have an initial idea about the application, in the first year of highschool, of the directives in the new syllabus concerning logic. 41 teachers have answered this questionnaire. First of all, we reproduce the answers that give us indications concerning the notions the teachers associate to logic:

Table 1 : What are the notions of logic that should be taught, for the teachers who have expressed themselves?

[2] I translate "l'entraînement à la logique fait partie intégrante des exigences des classes de lycée"

	Implication	Connectives And/Or	Proposition
Numbers of teachers having mentioned this notion	32	20	4

As we can see, the proposition, that is an essential element of an analysis of the mathematical language, isn't mentioned as a concept that should be a matter of learning. Concerning the implication, we have to ask them to specify if working on implication means working on a form of statement or working on the validity of certain inferences.

Other answers tell us about the existence of a teaching of notions of logic and on the influence of the changes in the syllabus over the teachers' practices:

Tableau 1 : Answers of the questionnaire about the teaching of notions of logic in the first year of highschool.

	YES	NO
3a) Did you work on notions of logic with your first year students before the new courses of 2009?[3]	21	16
3b) Do you work on notions of logic with your first year students since the new syllabus of 2009?	38	3
5a) In order to build a teaching allowing to reach the goals set by the syllabus, does your knowledge in mathematical logic seem enough for you?	30	11
5b) In order to organize a teaching that allows to reach the goals set by the official syllabus, did you find or easily conceived activities to offer your students?[4]	20	20

These answers show that beyond the syllabus, a majority of teachers think logic has its place in teaching mathematics. We can also observe a modification of the practices after the new syllabus has been set. Moreover, three quarters of the teachers having answered this questionnaire think they have enough theoretical knowledge to teach logic, but some of them consider this knowledge insufficient to allow them to conceive activities for their students. Moreover, a significant proportion of teachers still feel a lack of theoretical knowledge.

These first results obtained by the questionnaire need to be completed by interviews, in order to refine some of the answers. Moreover, we will complete the answers to the questionnaire and the interviews by some observation of class sequences, allowing us to witness the effective application of teaching notions of logic. This part of our work still being under process, we don't have any results to present yet.

[3] 4 teachers didn't teach in first year of highschool before 2009.
[4] 1 teacher didn't answer.

The works accomplished to answer theses research questions nurture the reflection about the training to offer teachers in the field of logic. We have been part of such training for 3 years, and we will introduce it more in detail, commenting the choices of contents we have made and the positions taken by the trainees during the debriefing of this training.

AN INITIATION TO LOGIC IN THE FRAMEWORK OF A CONTINUOUS TRAINING FOR TEACHERS.

The Institut de Recherche pour l'Enseignement des Mathématiques (IREM) at the University of Paris–Diderot proposed in 2011 a training course called “Introduction to logic” as part of the continuous training for teachers (this training course had already been organized in 2010 and reconducted in 2012). This course was led in collaboration with René Cori, professor in the logic team of the Paris Diderot University. Fifty teachers (the number of places was limited) enrolled in this course, fourty of them being effectively present. The training took place during three days of 6 hours each (two consecutive days in January, then one separated day a month later). One of the training goals is to bring the trainees knowledge in mathematical logic. This doesn’t mean lecturing about mathematical logic. It is all about teaching logic for teachers, a logic in context, at the service of mathematical activity. What is proposed is an anlysis and a critical look on mathematical language with whom teachers are already familiar. An important place is given to the notion of variable, that we will present as being characteristic of mathematical language in relation to the common language, and the multiple ways mathematics use to implicitely quantify their statements. The logical connective are then presented as operators on propositions, which means they allow, starting from one or two propositions, to “create” a third one. This syntactic aspect is separated from the semantic aspect broached by giving the truth tables of these connectives. The notions of tautology and propositions logically equivalent are defined and put in relation with the practices of reasoning. An important moment is dedicated to implication: establishing its truth table creates reactions. Then, it is essential to note that the negative of a conditional proposition is not a conditional proposition. We also discuss a long time about the implicite universal quantification associated to the formulation “if… then…”. We finally suggest few develoments on the study of theories. During the 2012 session, we have completed this theoretic program with a small conference on the natural deduction proposed by Paul Rozière, professor in the logic team of the Paris Diderot University.

We have offered a debriefing questionnaire at the end of the 2011 training. In this questionnaire, 30 over 35 teachers say they are in demande for a theoretical training in mathematical logic. From this point of view, they say they are satisfied by the chosen approach. 20 teachers consider sufficient the theoretical content proposed, and 12 of them would like to deepen the subject. These answers reinforce us in the choice of proposing develoments that are really part of the mathematical logic, that can give teachers a kind of culture of this mathematics branch, thus participating in modifying the idea they have of logic and the references they have to teach it.

Another important part of the training is a more practical aspect, asked for by the teachers having attented the 2010 session. We based ourselves on the study of the school textbooks, done amongst the “logic” work groupe of the Paris 7 IREM, that has been created after the

2010 training. Basing themselves on selected parts, the trainees did a critical analysis in small groups. At the end, we shared our work. This practical exercise allows to show the misunderstandings there can be about some notions, and is based on the taught theoretical components (this work is done after having spoken about variables, connectives and quantifiers). We also offered a moment during the third day, for the trainees who wished to present activites they have done in class, so that we could discuss about it. During the 2011 session debriefing, some teachers said they were unsatisfied concerning the small proportion of practical component of the training. Therefore, we have added to the 2012 session an intervention from teachers of the "logic" work group. They presented activities they have done in their classes. We also gave more time between the first two days and the last one, offering the trainee teachers to accompany them during the preparation of their sequence. The balance between theoretical and practical component was pointed by trainees as an important element of the 2012 training.

Thus, we have been able to get today to a form of training that seems satisfying to us, and that brings elements of theoretical knowledge as well as didactical ones. Directions of work are still to be developed amongst the group that coordinates the works of the different IREM groups concerning logic. Among these directions: to continue the propostions of activites in class and analyse the students' answers, to use all means to observe evolutions in theses answers, to detail a possible progress in the learnings of the three years of high school, to reinforce the learning of logic with situations that allow to exercise reasoning and with situations offering reformulations. We can also propose common activites about some notions of logic in French and in Mathematics.

CONCLUSION

The presence of notions of logic in the new 2009 mathematics syllabus for high school in France involves a certain novelty. Its introduction is an attempt to respond to recurring problems among students' expression and reasoning. But it is not evident how studying notions of logic can help overcome these difficulties. From a literature review we can see some difficulties students have to use some concepts of logic, especially in controlling their expression and their reasoning. We hypothesized that it is not an evidence for teachers how to teach logic of mathematics because they don't have clear mathematical references on logic while the knowledge on logic of mathematics must be particularly available due to the transversal nature of the study of concepts of logic. We can see some mistakes in new textbooks for high school in France, and this observation is consistent with our hypothesis. Teachers could have then difficulties to base their teaching on those documents, and be even more embarrassed that syllabuses are not very specific about what students have to learn. Some answers of teachers to a questionnaire we proposed show these difficulties. They also show that, for teachers who have answered, logic is more useful for reasoning than for expression. In continuous formation, we propose theoretical content in mathematical logic based on a naive approach which consist first to analyse the language the mathematicians use. We have now to continue our research to more precisely identify the practices and the needs of teachers and adapt the formation and the resources we can offer to them.

References

Arsac, G., & Durand-Guerrier, V. (2003) Méthodes de raisonnement et leurs modélisations logiques. Spécificité de l'analyse. Quelles implications didactiques ? *Recherches en didactique des mathématiques. 23/ 3*, 295-342.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59 (5),* 389-407.

Bridoux, S. (2009) Une séquence d'introduction des notions de topologie dans IR^N : de la conception à l'expérimentation. In A. Kuzniak & M. Sokhna (dir.), *Enseignement des mathématiques et développement: enjeux de société et de formation. Actes du Colloque Espace Mathématique Francophone,* 988-1001.

Chellougui, F. (2009) L'utilisation des quantificateurs universels et existentiels en première année d'université, entre l'implicite et l'explicite. *Recherches en didactique des Mathématiques, 29/2,* 123-154.

Chevallard, Y. (1985) *La transposition didactique. Du savoir savant au savoir enseigné.* Grenoble, France : La Pensée Sauvage éditions.

Dubinsky, E., Yiparaki, O. (2000) On students understanding of AE and EA quantification. *Research in Collegiate Mathematics Education IV, CBMS Issues in Mathematics Education 8,* 239-289.

Durand-Guerrier, V. & Arsac, G. (2008) Truth versus validity in mathematical proof. *ZDM The International Journal on Mathematics Education, 40 (3),* 373-384.

Lin, F.L., Hsieh F.J., Hanna, G., & de Villiers, M. (2009) *Proceedings of the ICMI Study 19 conference : Proof and Proving in Mathematics Education.* 140.122.140.1/~icmi19/files/Volume_1.pdf

Ravel, L. (2003) *Des programmes à la classe : étude de la transposition didactique interne. Exemple de l'arithmétique en Terminale S spécialité mathématique.* Thèse de l'Université Joseph Fourier Grenoble I.

Roh, K. (2010) An empirical study of students' understanding of a logical structure in the definition of the limit of a sequence via the -strip activity. *Educational Studies in Mathematics, 73,* 263-279.

Selden, A., & Selden, J. (1995) Unpacking the logic of mathematical statements. *Educational Studies in Mathematics 29,* 123-151.

Stylianides, A. J., & Ball, D. L. (2008). Understanding and describing mathematical knowledge for teaching: Knowledge about proof for engaging students in the activity of proving. *Journal of Mathematics Teacher Education, 11*, 307-332.